GENERAL CONSEQUENCES OF THE VIOLATED FEYNMAN SCALING

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1. Introduction The problem of scaling of the hadronic production cross sections represents an outstanding question in high energy physics especially for interpretation of cosmic ray data. A comprehensive analysis of the accelerator data /1/ leads to the conclusion for existence of breaked Feynman scaling. It was proposed that the Lorentz invariant inclusive cross sections for secondaries of a given type approache constent in respect to a breaked scaling variable x_s . Thus, the differential cross sections measured in accelerator energy $\sqrt{s_0}$ can be extrapolated to higher cosmic ray energies

$$\frac{E\, d^3\sigma}{dp_1 dp_t^2} = g(x_s, p_t) = f(x(s/s_0)^{\alpha/2}, p_t)\,(s/s_0)^{\alpha/2}\;, \qquad (1)$$

where $x = x(s/s_0)^{\alpha}$, $\alpha = 0.26$.

This assumption leads to some important consequences. In this paper w'l discuss the distribution of secondary multiplicity that follows from the violated Feynman scaling, using a similar method of Koba et al /2/.

2. Derivation of multiplicity distribution in the case of breaked Feynman scaling. The distribution of secondary particles in high energy hadron interactions is an object from the first to the recent observations with accelerator facilities. On the other hand, assuming Feynman scaling it was theoretically derived by Koba et al /2/ that assymptotically $\langle n\rangle\, P_n(s)$ is only a function of $n/\langle n\rangle$

$$P_n(s) = \sigma_n(s)/\sigma_{tot}(s) = \frac{1}{\langle n\rangle}\Psi(n/\langle n\rangle)\;, \qquad (2)$$

where $\sigma_n(s)$ is the cross section for multiplicity being n at CMS energy $\sqrt{s}$, $\langle n\rangle$ is the average multiplicity and $\Psi(z)$ is independent function of $\sqrt{s}$ exept through the variable $z=n/\langle n\rangle$. The shape of multiplicity distribution has been obtained in a variety of models with rather different theoretical inputs (uncorrelated cluster model /3/, geometrical models /4/, quark parton model /5/ and dual parton models /6,7/. The theoretical predictions for multiplicity distribution have been found to be approximately true from $\sqrt{s}$ =1.5 Gev up to ISR energy $\sqrt{s}$ =63 Gev /8/ where violation of Feynman scaling was observed. When studing the multiplicity distribution at the collider region at $\sqrt{s}$=540 Gev the KNO scaling does not necessarily holds for part of the phase space corresponding to higher multiplicities /9/. There is a clear indication of an increa-

sing high multiplicity tail. It causes that many of the original models have been amended to accomodate the observed scaling violation [10] by assuming that between the collider and ISR energies some new physical mechanism (rescattering [11], three gluon coupling [12]) characterized by higher multiplicity started becoming important [13]. We will examine what follows from breaking of Feynman scaling. Let us assume scaling on x_s for the distribution functions integrated over the transverse momentum

$$\tilde{g}^{(q)}(x_{s1},\dots,x_{sq}) = \int g^{(q)}(x_{s1},p_{t1},\dots,x_{sq},p_{tq})\, dp_{t1}^2\dots dp_{tq}^2 =$$

$$= \sqrt{\frac{s}{s_o}}^{\alpha}\tilde{f}\left(x_1\sqrt{\frac{s}{s_o}}^{\alpha},\dots,x_q\sqrt{\frac{s}{s_o}}^{\alpha}\right). \qquad (3)$$

There is needed only that the transwerse momentum is limited as $\sqrt{s}$ goes to imfinity. In eq. 3 are used functions which incorporate q particular semi-inclusive cross sections. We can derive the moments of multiplicity distribution in an analogical way to that of Koba et al [2]. Thus, for secondaries with rest mass m we set

$$\langle n(n-1)\dots(n-q+1)\rangle = \sum_n P_n(s)\, n(n-1)\dots(n-q+1) =$$

$$= \int g^{(q)}(x_{s1},p_{t1},\dots,x_{sq},p_{tq})\frac{dx_{s1}\, dp_{t1}^2}{\sqrt{x_{s1}^2+\frac{p_{t1}^2+m^2}{(s/4)^{1-\alpha}}}}\dots\frac{dx_{sq}\, dp_{tq}^2}{\sqrt{x_{sq}^2+\frac{p_{tq}^2+m^2}{(s/4)^{1-\alpha}}}} =$$

$$= 2\int\left\{\left[g^{(q)}(x_{s1},p_{t1},\dots,x_{sq},p_{tq})\cdot\ln\left(x_{s1}+\sqrt{x_{s1}^2+\frac{p_{t1}^2+m^2}{(s/4)^{1-\alpha}}}\right)\right]_0^{\frac{\sqrt{s}^{\alpha}}{2}}-\right.$$

$$-\int dx_{s1}\frac{\partial}{\partial x_{s1}}g^{(q)}(x_{s1},p_{t1},\dots,x_{sq},p_{tq})\cdot\ln\left(x_{s1}+\sqrt{x_{s1}^2+\frac{p_{t1}^2+m^2}{(s/4)^{1-\alpha}}}\right)\times$$

$$\times\ d^2p_{t1}\frac{dx_{s2}\, dp_{t2}^2}{\sqrt{x_{s2}^2+\frac{p_{t2}^2+m^2}{(s/4)^{1-\alpha}}}}\dots\frac{dx_{sq}\, dp_{tq}^2}{\sqrt{x_{sq}^2+\frac{p_{tq}^2+m^2}{(s/4)^{1-\alpha}}}} =$$

$$= \int\ln\frac{(s/4)^{1-\alpha}}{(p_{t1}^2+m^2)}\left[g^{(q)}(0,p_{t1},x_{s2},p_{t2},\dots,x_{sq},p_{tq}) + \frac{W}{\ln\frac{(s/4)^{1-\alpha}}{(p_{t1}^2+m^2)}}\, d^2p_{t1}\right.$$

$$\times\frac{dx_{s2}\, d^2p_{t2}}{\sqrt{x_{s2}^2+\frac{p_{t2}^2+m^2}{(s/4)^{1-\alpha}}}}\dots\frac{dx_{sq}\, d^2p_{tq}}{\sqrt{x_{sq}^2+\frac{p_{tq}^2+m^2}{(s/4)^{1-\alpha}}}} \qquad (4)$$

where the integral $W = \int dx_{s1}\frac{\partial}{\partial x_{s1}}\, g^{(q)}(x_{s1},p_{t1},\dots,x_{sq},p_{tq})\ln\left(x_{s1}+\sqrt{x_{s1}^2+\frac{p_{t1}^2+m^2}{(s/4)^{1-\alpha}}}\right)$ converges. After integration of eq. 4 we obtain

$$\langle n(n-1)\ldots(n-q+1)\rangle=\int\left[\ln\frac{(s/4)^{1-\alpha}}{p_{t1}^2+m^2}\ldots\ln\frac{(s/4)^{1-\alpha}}{p_{tq}^2+m^2}\left[g^{(q)}(0,p_{t1},\ldots 0,p_{tq})\right.\right.$$

$$\left.\left.+\ O\left(\frac{1}{\ln s^{1-\alpha}}\right)\right]dp_{t1}^2\ldots dp_{tq}^2=\tilde{g}^{(q)}(0,\ldots,0)(\ln s^{1-\alpha})^q+O((\ln s^{1-\alpha})^{q-1}),$$

where $O((\ln s^{1-\alpha})^{q-1})$ means terms that at most go like $(\ln s^{1-\alpha})^{q-1}$. Consequently the same asymptotic behaviour has not only the mean value of any q-order polynomial of n but the mean value of n as well. Taking into account eq.3 we can set

$$\int_0^\infty n^q\cdot p_n(s).dn=\sqrt{\frac{s}{s_0}}^{\alpha}f^{(q)}(0,\ldots,0)\ (\ln s^{1-\alpha})^q+O(\ln s^{1-\alpha})^{q-1})\ . \tag{5}$$

Dividing eq.5 by $\sqrt{\frac{s}{s_0}}^{\alpha}(\ln s^{1-\alpha})^q[\tilde{f}^{(1)}(0)]^q$ we obtain

$$\int_0^\infty z.p_{z(n)}(s).\tilde{f}^{(1)}(0)\sqrt{\frac{s}{s_0}}^{\alpha}\ln s^{1-\alpha}.dz=\frac{\tilde{f}^{(q)}(0,\ldots,0)}{[\tilde{f}^{(1)}(0)]^q}+O\left(\frac{1}{\sqrt{s}^{\alpha}\ln s^{1-\alpha}}\right), \tag{6}$$

where

$$z=\frac{n}{\tilde{f}^{(1)}(0)\sqrt{\frac{s}{s_0}}^{\alpha}\ln s^{1-\alpha}} \tag{7}$$

We assume that the function

$$\tilde{f}^{(1)}(0)\sqrt{\frac{s}{s_0}}^{\alpha}\ln s^{1-\alpha}p_{z(n)}(s)=\Psi(z)+O\left(\frac{1}{\sqrt{s}^{\alpha}\ln s^{1-\alpha}}\right)$$

is determined uniquely by the moments (5). Thus, to the highest order in $\sqrt{s}^{\alpha}\ln s^{1-\alpha}$ we have the follwing breaked scaling result

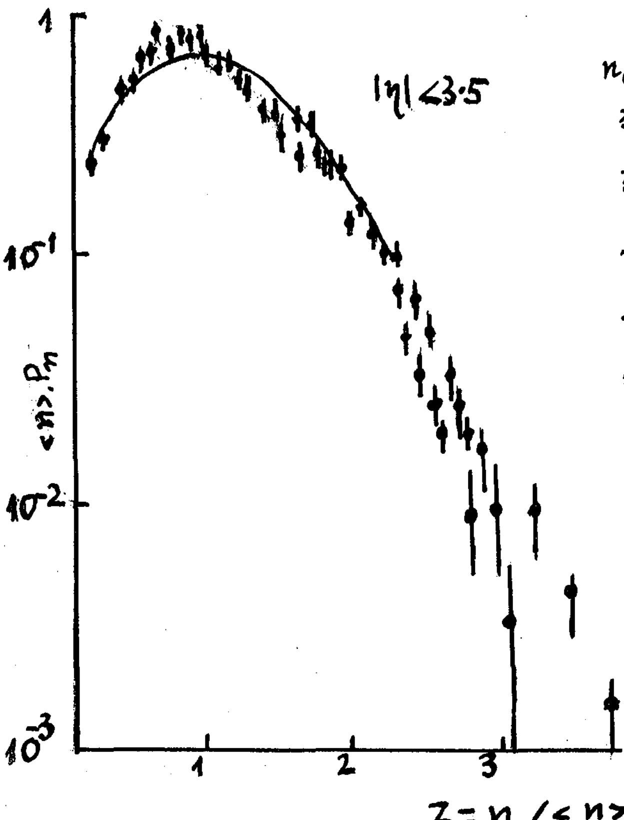
$$P_n(s)=\frac{1}{\langle n\rangle}\Psi(z)\ +\ O\left(\frac{1}{\langle n\rangle^2}\right)\quad, \tag{8}$$

where the mean multiplicity as function of $\sqrt{s}$ is

$$\langle n\rangle\simeq\tilde{f}^{(1)}(0)\sqrt{\frac{s}{s_0}}^{\alpha}\ln s^{1-\alpha}\quad. \tag{9}$$

<u>3. Comparison of the breaked Feynman scaling results for the multiplicity distribution with experimental data in accelerator energy range.</u> In figure 1 we have compared the multiplicity distribution function specified by the semi-inclusive cross sections from FNAL which ate published in the paper of Kafka et al /14/ for multipartical production up to above two times larger than the mean value of multiplicity. It is seen that the distribution of relative multiplicity scales at least for the range of not very large multiplicities which are responsible for the typical events in cosmic ray experiments.

As far as the average multiplicity is concerned the assumption for breaking of Feynman scaling gives a good agreement with the acceleration observations in wide energy range up to 10^5Gev. The energy dependence of mean multiplicity according eq.9 is compared in figure 2 with accelerator data taken from Carlson /15/.



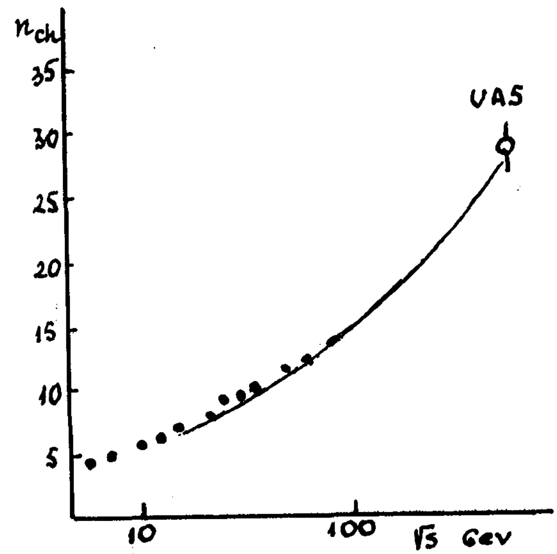


Fig.2 Energy dependence of the mean multiplicity of charged secondaries. Experimental data are taken from[16], the full line is after eq.9 .

Fig.1 Comparison of our parametrisation of the multiplicity distribution (made on the basis of FNAL data) with SPS data for $|\eta|<3.5$. The latter are representative of the shape of the full distribution.

We can conclude that for the purposes of cosmic ray investigation scaling of multiplicity distribution derived from breaked Feynman scaling can be assumed in order to prescribe the semi-inclusive cross sections at very high energies.

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